

Inflation and Alternatives with Blue Tensor Spectra

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We study the tilt of the primordial gravitational waves spectrum. A hint of blue tilt is shown from analyzing the BICEP2 and POLARBEAR data. Motivated by this, we explore the possibilities of blue tensor spectra from the very early universe cosmology models, including null energy condition violating inflation, inflation with general initial conditions, and string gas cosmology, etc. For the simplest G-inflation, blue tensor spectrum also implies blue scalar spectrum. In general, the inflation models with blue tensor spectra indicate large non-Gaussianities. On the other hand, string gas cosmology predicts blue tensor spectrum with highly Gaussian fluctuations. If further experiments do confirm the blue tensor spectrum, non-Gaussianity becomes a distinguishing test between inflation and alternatives.

I. INTRODUCTION

Inflation [1, 2] is the leading paradigm for the very early universe cosmology. Inflation has been proposed to explain the horizon, flatness and monopole problems in the standard hot big bang cosmology, and almost all the predictions of the simplest inflation model have now been tested. The observational tests of inflation includes

- Coherent and nearly scale invariant power spectrum of density perturbations. The power spectrum of the simplest slow roll inflation is [3]

$$P_\zeta = \frac{H^2}{8\pi^2\epsilon M_p^2} \simeq 2.43 \times 10^{-9} . \quad (1)$$

- A small tilt of the scalar power spectrum.

$$n_s - 1 = -2\epsilon - \eta \simeq 0.96 , \quad (2)$$

where $\eta \equiv \dot{\epsilon}/(H\epsilon)$ is the slow roll parameter defined from expansion. Now $n_s \geq 1$ is ruled out under the assumptions of simplest inflation models.

- Nearly Gaussian density fluctuations. The non-Gaussianities of the density fluctuations are tightly constrained at

$$f_{\text{NL}}^{\text{local}} = 2.7 \pm 5.8 , \quad f_{\text{NL}}^{\text{equil}} = -42 \pm 75 \quad (3)$$

for the local shape and equilateral shape non-Gaussianities respectively. Those numbers indicate that, non-Gaussian components of the primordial fluctuations, even if exist, have to be at least 3~4 orders of magnitudes smaller than the Gaussian component.

- Gravitational waves. The recent BICEP2 experiment reports an over 5σ detection of gravitational waves [4], with tensor to scalar ratio

$$r = 0.20^{+0.07}_{-0.05} (1\sigma\text{CL}) . \quad (4)$$

This corresponds to a gravitational wave fluctuation amplitude

$$P_T = \frac{2H^2}{\pi^2 M_p^2} = 4.8 \times 10^{-10} . \quad (5)$$

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Despite of the great success of inflation, there are still a few outstanding challenges for theorists and experimentalists.

On the theoretical side, large field inflation is now favored. However, large field inflation is hard to construct from the effective field theory, and stringy UV completion points of view. The UV completion of inflation has long suffered from an η -problem [5], in which the mass of the inflaton is theoretically too large to allow enough e-folds of inflation. However, with the current data, a more serious ϵ -problem emerges – the observed energy scale of inflation is too high for an effective field theory or stringy model building to be under control. For single field slow roll inflation, at every e-fold, the inflaton rolls a distance of order $0.1M_p$. In perturbative string theory, this field motion per e-fold is comparable with, or greater than the string scale M_s . As a result, one can no longer safely globally expand the inflaton field and ignore non-renormalizable terms. More discussions and a local reconstruction of the inflationary potential can be found in [6]

On the observational side, there is yet another (and maybe the last in the foreseeable future, unless nature is so kind as to imprint other relics on the CMB sky or in the large scale structure) test for inflation which is possible in light of BICEP2, but not yet achieved – the tilt of the tensor power spectrum. The simplest inflation models predict a consistency relation between n_T and r as

$$n_T = -\frac{r}{8} = -0.025. \quad (6)$$

Currently the data has not been good enough to test n_T precisely. However, there are a lot of ongoing and upcoming experiments in the near future [7–11], measuring r at different scales, which provides a possibility for a precise measurement of n_T .

In this paper, we shall explore the possibility of blue n_T . In Section II, the bound on n_T is derived the BICEP2 [4] and POLARBEAR [8] data. The string gas cosmology, null energy condition (NEC) violating inflation, and general initial condition are addressed in Sections III, IV and V respectively. In Section VI, a few other possibilities are discussed, including external sources for tensor modes, modified gravity and matter bounce. We conclude in Section VII.

II. HINTS FOR BLUE TENSOR SPECTRA

With the current BICEP2 and POLARBEAR polarization data, the questions we would like to address in this section are : 1) what is the current bound of the tilt of the primordial gravitational wave spectrum n_T ; 2) how n_T correlates with the tensor-to-scalar ratio r .

In order to study this correlation, we are performing the analysis of the BB power spectrum against the data of BICEP2 and POLARBEAR, where the spectrum is calculated by the public code CLASS [12]. We are focusing on the BB power spectrum here due to the fact that the character of the primary gravitational waves contains amplitude and the tilt which can be represented by the two parameters, i.e. n_T and r . For the other parameters, such as the primordial scalar spectrum, it is not directly related to the BB power spectrum; the value of the amplitude and tilt of the scalar adiabatic perturbation is set by $A_s = 2.43 \times 10^{-9}$ and $n_s = 0.962$ [13]. Notice that the pivot scale of the scalar and tensor spectrum are both set at $k = 0.002 \text{ Mpc}^{-1}$ or $k = 0.01 \text{ Mpc}^{-1}$ in our analysis¹. And other cosmological parameters to calculate the CMB spectrum are chosen as used by BICEP2.

The relation between r and n_T is illustrated in fig. 1. At $r = 0.18$, the flat tensor spectrum gives a good fit to the data, which is consistent with the result of the BICEP2 [4]. With the fixed ratio r at pivot scale 0.002 Mpc^{-1} , by increasing or decreasing the tilt n_T by 0.2, it starts to be ruled out by the current data. If we consider the smaller tensor-to-scalar ratio, indicated from the right panel of fig. 1, a blue tensor spectrum is required by the BICEP2 data.

A more precise correlation of n_T and r is illustrated in fig. 2, fig. 3 and fig. 4, which choose different pivot scales. Fig. 2 with pivot scale at 0.002 Mpc^{-1} is same as the analysis of BICEP2 paper [4], while in fig. 3 and fig. 4 the pivot scale is chosen at the scale where observation is roughly made, $k = 0.01 \text{ Mpc}^{-1}$. In the left panel of the figures, the full data set of BICEP2 and POLARBEAR are employed, which shows a preference for the blue tensor spectrum. And the $1\text{-}\sigma$ confidence region is in $n_T > 0$. Clearly from the χ^2 plots of fig. 4, the best fit point is at $r = 0.178$ and $n_T = 1.05$.

On the other hand, the systematic uncertainties can come from the template subtraction of the polarization maps; and the first five data bins are mainly originated from the primordial gravitational waves but the other bins have more contributions or contaminations. By considering these, we introduce the second method to deal with the data. The first five data bins are computed by χ^2 method and the others are treated as an upper bound. The result showing in

¹ Thus when comparing our r parameter with the BICEP2 value, a factor should be multiplied: $r_{\text{BICEP2}} \simeq r \times 2.43/2.20 \simeq 1.10r$, where the ratio comes from that BICEP2 use pivot scale $k = 0.05 \text{ Mpc}^{-1}$ for the scalar power spectrum. For example, the central value $r_{\text{BICEP2}} = 0.2$ corresponds to $r \simeq 0.18$ in our analysis.

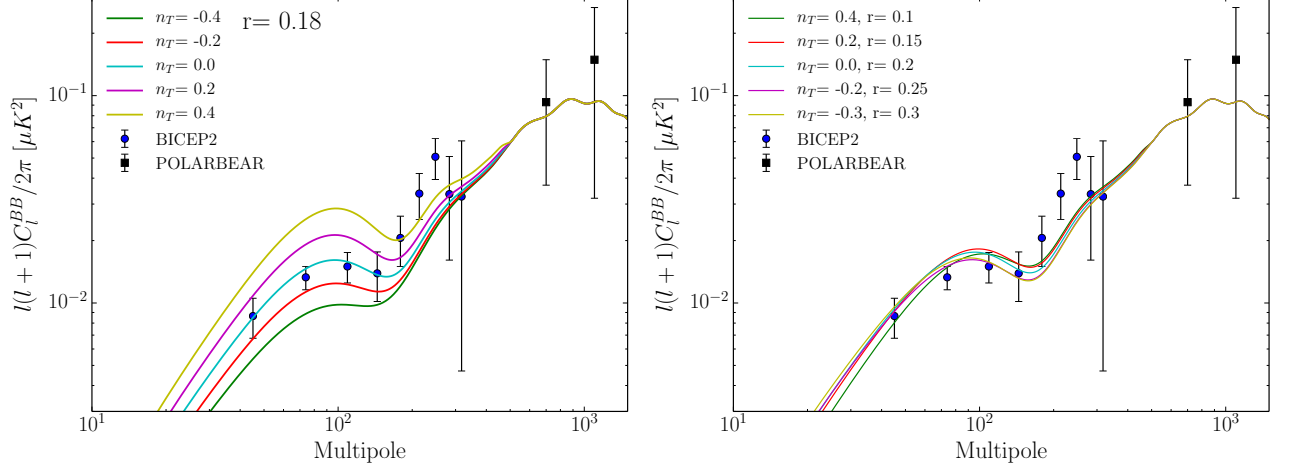


FIG. 1. The simulated BB power spectrum with different $r_{0.002}$ and n_T . Left panel: $r_{0.002}$ is fixed and variation of n_T is illustrated. Right panel: some sets of $r_{0.002}$ and n_T are chosen to get reasonable fit against data.

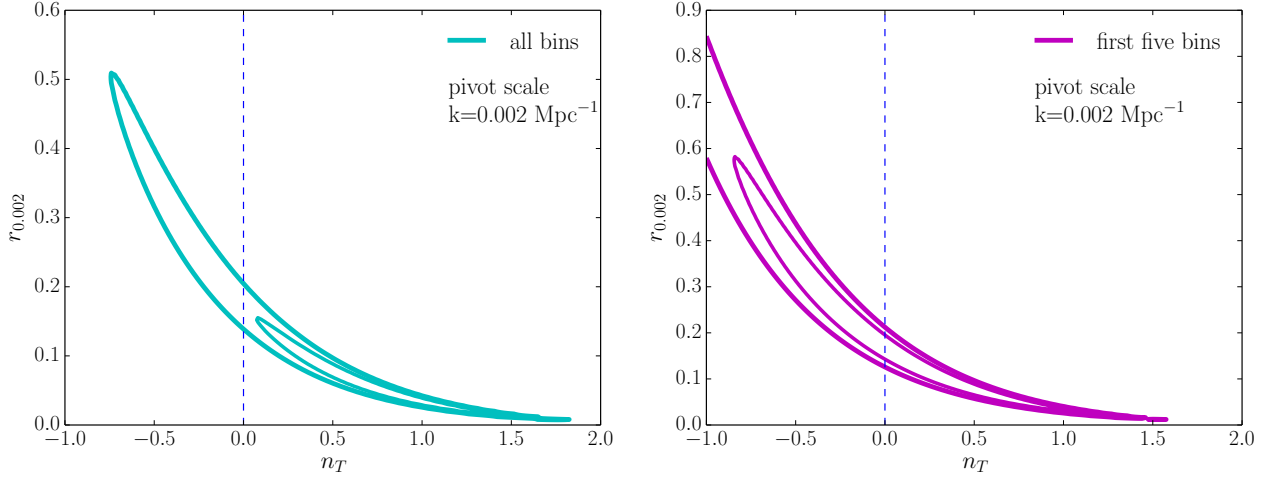


FIG. 2. Contour plot of $r_{0.002}$ vs n_T to fit against the data. Left panel: fit the data of the BICEP2 and POLARBEAR. Right panel: fit the first five data bins of BICEP2 and treat the others as upper bound. Note that the strong correlation between n_t and $r_{0.002}$ is an artifact from the choice of pivot scale. We have chosen $k = 0.002 \text{ Mpc}^{-1}$ to match with BICEP2 conventions. However, the BICEP2 experiments measures $k \sim 0.01 \text{ Mpc}^{-1}$. Thus given a similar tensor spectrum at $k \sim 0.01 \text{ Mpc}^{-1}$, the large tilt modifies the $r_{0.002}$ value significantly.

the right panels of fig. 2 and fig. 3 is that 1- σ confidence region is shifted and the central value is more close to the flat spectrum $n_T = 0$ than the analysis considering all the bins.

Before moving to theoretical possibilities for blue tensor spectra, a few comments about data aspects are in order:

- A blue tensor tilt helps to reconcile the tension between Planck and BICEP2 results ². Note that currently, the conclusion of small r from Planck comes from the temperature map, with $l \lesssim 100$. On the other hand, the BICEP2 results comes from $50 \lesssim l \lesssim 100$. A blue tilt of tensor spectrum suppresses the $l \lesssim 50$ part of the TT spectrum and thus reconciles the tension between Planck and BICEP2.
- A blue tensor tilt helps resolving the anomaly of small l suppression [15]. The TT power spectrum shows a

² This point is added in v2 of this paper. A similar viewpoint [14] has already appeared before our revision.

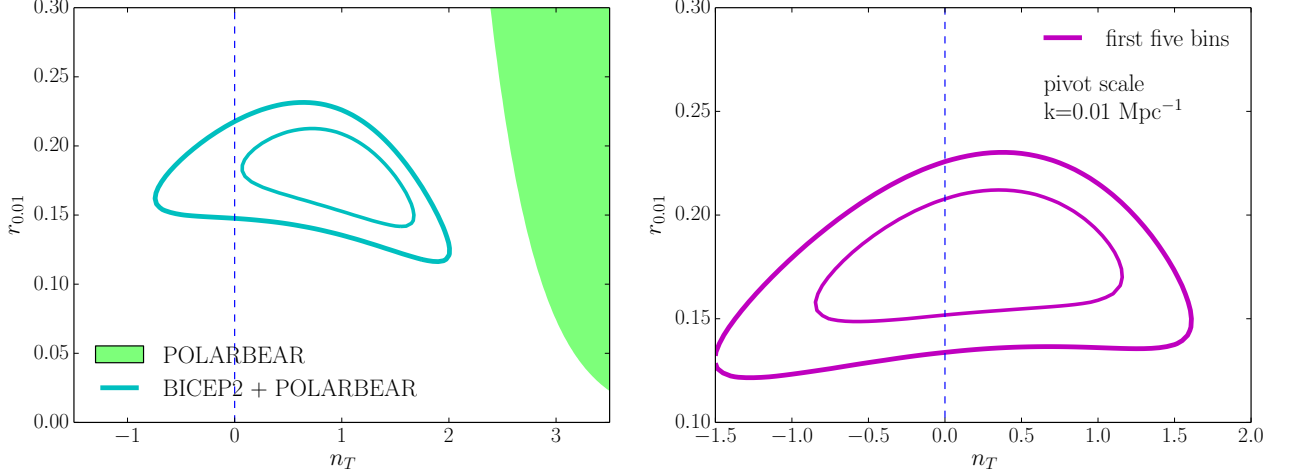


FIG. 3. Contour plots of $r_{0.01}$ vs n_T to fit against the data. Left panel: fit the data of the BICEP2 and POLARBEAR. The green region is excluded by POLARBEAR. Right panel: fit the first five data bins of BICEP2 and the others are upper bound. The pivot scale is chosen as $k = 0.01 \text{ Mpc}^{-1}$, which is approximately the same scale as the BICEP2 detection is made.

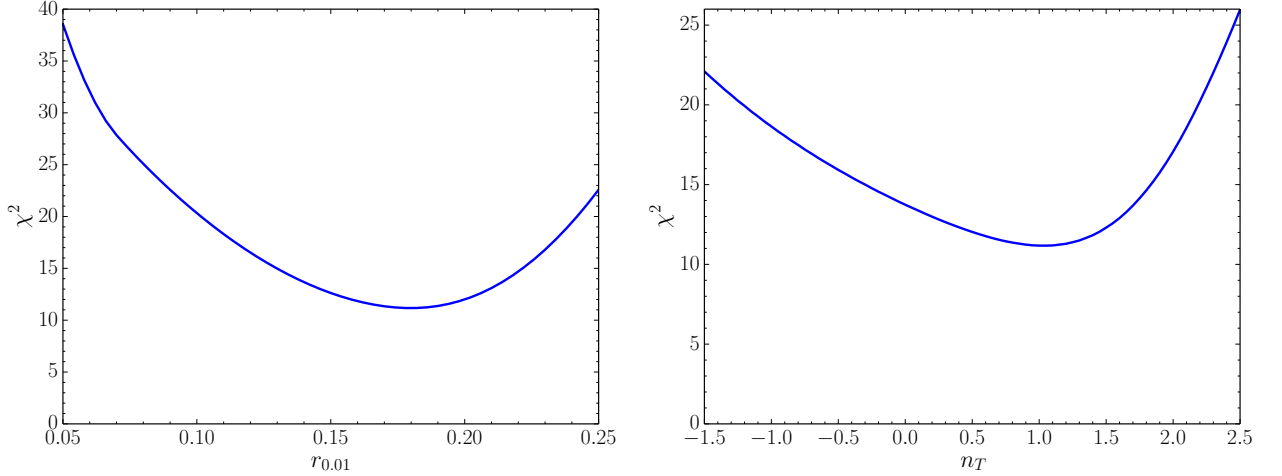


FIG. 4. χ^2 is a function of r and of n_T , which fits the data of BICEP2 and POLARBEAR with the pivot scale 0.01 Mpc .

deficit of power at $l \leq 40$, of $5\% \sim 10\%$. If the tensor mode contributes to TT more at larger l (i.e. blue tilt), it helps to resolve the missing power anomaly.

- It is conventionally believed that the POLARBEAR detection of B-modes (and earlier, the cross correlation detected by SPTpol [16]) comes from lensing. This is true only if the tensor spectrum is not very blue. When $n_T \gtrsim 2.5$, the primordial gravitational waves dominate over lensing at high l . Thus on the one hand, if n_T is indeed very blue, the detection from SPTpol and POLARBEAR may include considerable amount of primordial gravitational waves contribution; and on the other hand, those experiments puts tight constraint on $n_T \gtrsim 2.5$. To see this explicitly, the exclusion curve from POLARBEAR is plotted in the left panel of Fig. 3. It remains interesting to see if the SPTpol cross correlation puts similar or tighter bound on n_T .
- Although the current bound on n_T is far from testing the consistency relation of inflation, it is already informative to disfavor scenarios with very blue or red spectrum. For example, a sharp pulse of gravitational waves with rapid decaying tail towards both ends ($n_T < -1.5$ or $n_T > 2.0$) are disfavored by the current data.
- Very blue tensor tilt cannot last long. Assuming the running of n_T is not significant, then for $n_T = 1$, it takes about 23 e-folds to bring the tensor mode to be non-perturbative ($P_h \sim 1$). Those non-perturbative tensor modes

forms primordial black holes, which is constrained from current observations. For $n_T = 2$, it takes about 12 e-folds. Thus assuming

$$\alpha_T \equiv \frac{d \log n_T}{d \log k} \ll 1, \quad (7)$$

we get $n_T < 0.38$ for 60 e-folds of inflation and $n_T < 0.46$ for 50 e-folds of inflation. Nevertheless, if n_T is large, α_T may be large as well. If α_T is negative, it relaxes the above constraint.

III. STRING GAS COSMOLOGY

Before a survey of inflationary possibilities for the blue tensor spectra, let us mention alternatives to inflation. This is because the first prediction of a slightly blue n_T comes from the string gas cosmology [17–20] (see [21] for a recent discussion in light of BICEP2 data)³. On the contrary, for the other models that we shall show below, though they have the possibility to tune the tensor spectrum to be blue, the blueness is not a firm prediction.

In string gas cosmology, the universe was in a string Hagedorn phase before expansion starts. It is conjectured that due to T-duality, and the huge specific heat in the string Hagedorn phase, the universe should stay at a nearly constant temperature (Hagedorn temperature T_H) for a long time, until the string winding modes decays and allow the expansion of the universe (which is also a possible explanation of three large spatial dimensions).

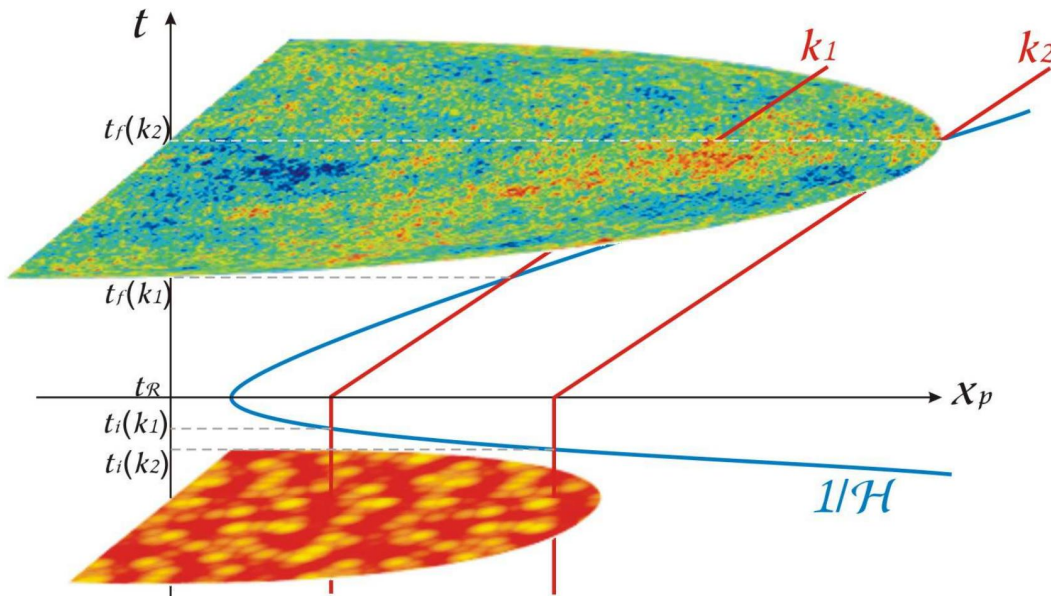


FIG. 5. String gas cosmology. The universe starts from a string Hagedorn phase and a phase transition brings the universe into radiation dominated.

The density and tensor fluctuations in string gas cosmology are thermal fluctuations. Those fluctuations were in causal contact in the string Hagedorn phase, and get frozen on super-Hubble scales because of the rapid shrinking of the Hubble horizon (Fig. 5). The scalar and tensor fluctuations can be calculated as [18–20]

$$P_\Phi(k) = \left(\frac{l_p}{l_s}\right)^4 \frac{T(k)}{T_H} \frac{1}{1 - T(k)/T_H}, \quad (8)$$

where Φ is the Newtonian potential. The scalar spectral index is

$$n_s - 1 = \frac{d \ln(1 - T(k)/T_H)}{d \ln k}. \quad (9)$$

³ The Ekpyrotic scenario [22](see [23] for a detailed analysis of tensor modes) also predicts a blue tensor spectrum with $n_T = 2$. This is consistent with our n_T bound if all the BICEP2 data points are fitted. However, the amplitude of the tensor mode is exponentially suppressed on cosmological scales, which cannot be $r = 0.2$ at $k = 0.002 \text{ Mpc}^{-2}$. There is a similar issue for the bouncing Galileons [24].

From the string gas picture, one expects that $T(k)$ is an increasing function of k and thus string gas cosmology has red scalar spectrum.

The tensor power spectrum is

$$P_h(k) = \left(\frac{l_p}{l_s}\right)^4 \frac{T(k)}{T_H} \left(1 - \frac{T(k)}{T_H}\right) \ln^2 \left[\frac{1 - T(k)/T_H}{l_s^2 k^2} \right]. \quad (10)$$

The tensor to scalar ratio is

$$r_{h\Phi} = \left(1 - \frac{T(k)}{T_H}\right)^2 \ln^2 \left[\frac{1 - T(k)/T_H}{l_s^2 k^2} \right]. \quad (11)$$

On the other hand, the tensor to scalar ratio measured by BICEP2 is between h and the comoving curvature perturbation ζ . This results in a conversion factor. On super-Hubble scales,

$$\zeta \simeq \left(1 + \frac{\mathcal{H}^2}{\mathcal{H}^2 - \mathcal{H}'}\right) \Phi, \quad (12)$$

where \mathcal{H} is the comoving Hubble parameter and prime denotes derivative with respect to the comoving time. Assuming the Hagedorn phase is fast enough to the radiation dominated era, the factor can be calculated as

$$\zeta \simeq \frac{3}{2} \Phi. \quad (13)$$

Thus the tensor to scalar ratio using ζ is

$$r = \frac{4}{9} r_{h\Phi} = \frac{4}{9} \left(1 - \frac{T(k)}{T_H}\right)^2 \ln^2 \left[\frac{1 - T(k)/T_H}{l_s^2 k^2} \right]. \quad (14)$$

Inserting the COBE normalization $P_\zeta = 2.43 \times 10^{-9}$ and the BICEP2 central value $r = 0.2$, the string scale is derived as $M_s \simeq 10^{-3} M_p$.

The tensor spectral index is

$$n_T = -(n_s - 1) \left(2 \frac{T(k)}{T_H} - 1\right) \simeq -(n_s - 1). \quad (15)$$

Thus string gas cosmology predicts blue tensor spectra.

It is also worth to mention that string gas cosmology produces highly Gaussian density perturbations (unless the string scale is near TeV scale, which is not preferred from current data). The f_{NL} estimator for string gas cosmology can be calculated as [25]⁴

$$f_{\text{NL}} \sim \left(\frac{l_s}{l_p}\right) \times 10^{-30} \frac{k}{k_0}, \quad (16)$$

where k_0 corresponds to the scale of our present observable universe. With the detection of r , the string scale is known in string gas cosmology. As a result, $f_{\text{NL}} \sim 10^{-27} k/k_0$. This is orders-of-magnitude smaller than the observational bound.

As we shall see, the inflationary candidates with blue n_T , as far as we consider, produce considerable amount of non-Gaussianities. Thus in case a blue tensor tilt is detected, non-Gaussianity should be the next test (and maybe already much better constrained than now by the time of n_T measurement) to distinguish between string gas and inflationary models.

IV. INFLATION: VIOLATION OF NEC

In standard Einstein-Hilbert gravity, the amplitude of the tensor modes is determined by the energy scale of inflation. And thus for the case of blue n_T , super inflation is needed, with null energy condition (NEC) violation [26].

⁴ Note that only the non-Gaussianity near the horizon-crossing is calculated in [25]. The Hubble-scale gravitational non-linearity may also introduce some non-Gaussianities, typically suppressed by slow varying parameters.

It has been debated for a long time that if the cosmological (inflation, alternatives or dark energy) background violates NEC, the scalar sector of the inflationary perturbations should become a ghost. However, this situation has been changed since Galileons are introduced to inflation. Here we shall explore the parameter space of the simplest G-inflation scenario, with slight generalization from the explicit model with an exponential potential in [27] into a general slow roll functional dependence.

We start from the action of G-inflation

$$\mathcal{L}_\phi = K(\phi, X) - G(\phi, X)\Box\phi, \quad (17)$$

where $X \equiv -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$. Here we restrict our attention to the models with

$$K(\phi, X) = -X + \frac{X^2}{2M^3\mu}, \quad G(\phi, X) = g(\phi)X. \quad (18)$$

A special class of de Sitter solution is worked out in [27]:

$$3M_p^2 H^2 = -K, \quad K_X + 3gH\dot{\phi} = 0. \quad (19)$$

We shall consider small derivation of the de Sitter solution (19). For this purpose, define small parameters

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \epsilon_\phi \equiv -\frac{\ddot{\phi}}{H\dot{\phi}}, \quad \epsilon_g \equiv M_p \frac{g_\phi}{g}, \quad \epsilon_\mu \equiv \frac{\mu}{M_p}. \quad (20)$$

When expanding around the de Sitter solution, the ϵ , ϵ_ϕ , and ϵ_g parameters should indeed be small. The consistency for the smallness of the ϵ_μ parameter shall be checked later.

In terms of those slow roll parameters, and near the de Sitter solution, the order of magnitude of the following quantities can be estimated:

$$XK_X \sim K\mathcal{O}(\epsilon_\mu), \quad XgH\dot{\phi} \sim K\mathcal{O}(\epsilon_\mu), \quad G_\phi X \sim K\mathcal{O}(\epsilon_g\epsilon_\mu). \quad (21)$$

The second order action of the scalar perturbations can be written as

$$S^{(2)} = \frac{1}{2} \int d\tau d^3x z^2 [\mathcal{G}(\mathcal{R}')^2 - \mathcal{F}(\partial_i \mathcal{R})^2], \quad (22)$$

where with the above slow roll approximation, the \mathcal{F} and \mathcal{G} functions can be written as (for unapproximated definition, see [27])

$$\mathcal{F} = -\frac{1}{3}K_X + \mathcal{O}(\epsilon_\mu^2), \quad \mathcal{G} = 2XK_{XX} + \mathcal{O}(\epsilon_\mu). \quad (23)$$

Thus the scalar perturbations are stable in general for $g(\phi)$ satisfying the above slow roll conditions. The power spectrum and spectral index of the scalar sector can be calculated as

$$P_{\mathcal{R}} = \frac{Q}{4\pi^2}, \quad n_s - 1 = -2\epsilon\mathcal{C}, \quad (24)$$

where

$$Q = \frac{K^2}{18M_p^4 X} \sqrt{\frac{\mathcal{G}}{\mathcal{F}^3}}, \quad \mathcal{C} = -\frac{3}{2} \frac{KK_{XX}}{K_X^2} + \mathcal{O}(\epsilon_\mu). \quad (25)$$

Note that $K < 0$ and $K_{XX} > 0$ near the de Sitter solution. Thus $\mathcal{C} > 0$.

The gravity sector is not modified. Thus the tensor mode has the conventional spectrum

$$P_T = \frac{2H^2}{\pi^2 M_p^2}. \quad (26)$$

The tensor spectral index is

$$n_T = -2\epsilon \quad (27)$$

Note that for the tensor mode to be blue, we need $\epsilon < 0$. This is indeed possible. However, it is worth noting that from (24),

$$n_s - 1 > 0 . \quad (28)$$

In other words, near the de Sitter solution (19), the scalar and tensor modes tilt towards the same direction – both red spectra or both blue spectra. Note that a blue scalar spectrum is not favored by observations. However, in more general cosmological models, for example, with a different number of neutrinos, a blue scalar spectrum is not yet ruled out.

On the other hand, we are not sure at this point if the same direction of tilt for scalar and tensor spectra is a general feature of G-inflation (and generalized Galileons [28–30]), or there exists unexplored models with different tilt of two sectors.

Also, one has to note the sound speed of the scalar sector of perturbations is

$$c_s^2 = \frac{\mathcal{F}}{\mathcal{G}} = -\frac{1}{6} \frac{K_X}{K_{XX}} \sim 0.03 \quad (29)$$

This would induce an equilateral non-Gaussianity $|f_{\text{NL}}^{\text{equil}}| \sim 30$. This is consistent with the current *Planck* bound, and on the other hand accessible in the future.

It is worth mentioning that the small c_s and large $|f_{\text{NL}}^{\text{equil}}|$ are not a coincidence in this particular toy model, but should be a rather model-independent statement (unless fine tuned). To violate NEC at the background level while keeping the perturbations stable, the relevant Galileons Lagrangian at the energy scale of inflation should be highly non-linear. Non-linear self-coupling introduces equilateral non-Gaussianity. Experiments in the near future would reduce the bound for $|f_{\text{NL}}^{\text{equil}}|$ (or detection) and examine the possibility of Galileons and NEC violation.

Finally, one can check that when $r \simeq 0.2$, $\epsilon_\mu \simeq 0.03$. Thus the smallness of ϵ_μ is consistent with data.

V. INFLATION: GENERAL INITIAL CONDITIONS

It has been an open question if the inflationary (scalar and tensor) perturbations originates from a Bunch-Davies (BD) vacuum, or special care needs to be taken for the non-BD initial conditions. One motivation for choosing non-BD initial conditions, for example, is the transPlanckian problem of inflation [31, 32] (see [33] for a recent review)⁵.

Inflation requires UV completion. During inflation, the perturbations originates from the UV completion scale, for example, Planck scale, before they expand and cross the Hubble horizon. Note that the Planck scale could have been replaced by some lower scales, for example, the string scale. With those energy scales for new physics, we can no longer make sure that the inflationary perturbations were in their lowest energy states before they got stretched by the cosmic expansion. This situation is illustrated in Fig. 6.

In the context of tensor perturbations [35] (for a related recent study, see also [36]), the action of tensor mode is

$$S = \frac{M_p^2}{8} \int \frac{d^3k}{(2\pi)^3} d\tau \, a^2 \left(\gamma_j^{i'} \gamma_i^{j'} - k^2 \gamma_j^i \gamma_i^j \right) . \quad (30)$$

Decomposing the tensor modes by polarization, we have

$$\gamma_{ij}(\mathbf{k}) = \frac{\sqrt{2}}{M_p} \left[\gamma_+(\mathbf{k}) e_{ij}^+(\mathbf{k}) + \gamma_\times(\mathbf{k}) e_{ij}^\times(\mathbf{k}) \right] , \quad (31)$$

as a result, the action contains two copies of modes

$$S = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} d\tau \, a^2 \left[(\gamma_+' \gamma_+' - k^2 \gamma_+ \gamma_+) + (\gamma_\times' \gamma_\times' - k^2 \gamma_\times \gamma_\times) \right] . \quad (32)$$

For each mode, one can impose a different initial condition from BD.

The γ (collectively denote γ_+ and γ_\times) field can be quantized as

$$\gamma_k = v_k a_k + v_k^* a_{-k}^\dagger \quad (33)$$

⁵ There are also other possibilities. For example, thermal fluctuations as initial condition of inflation [34].

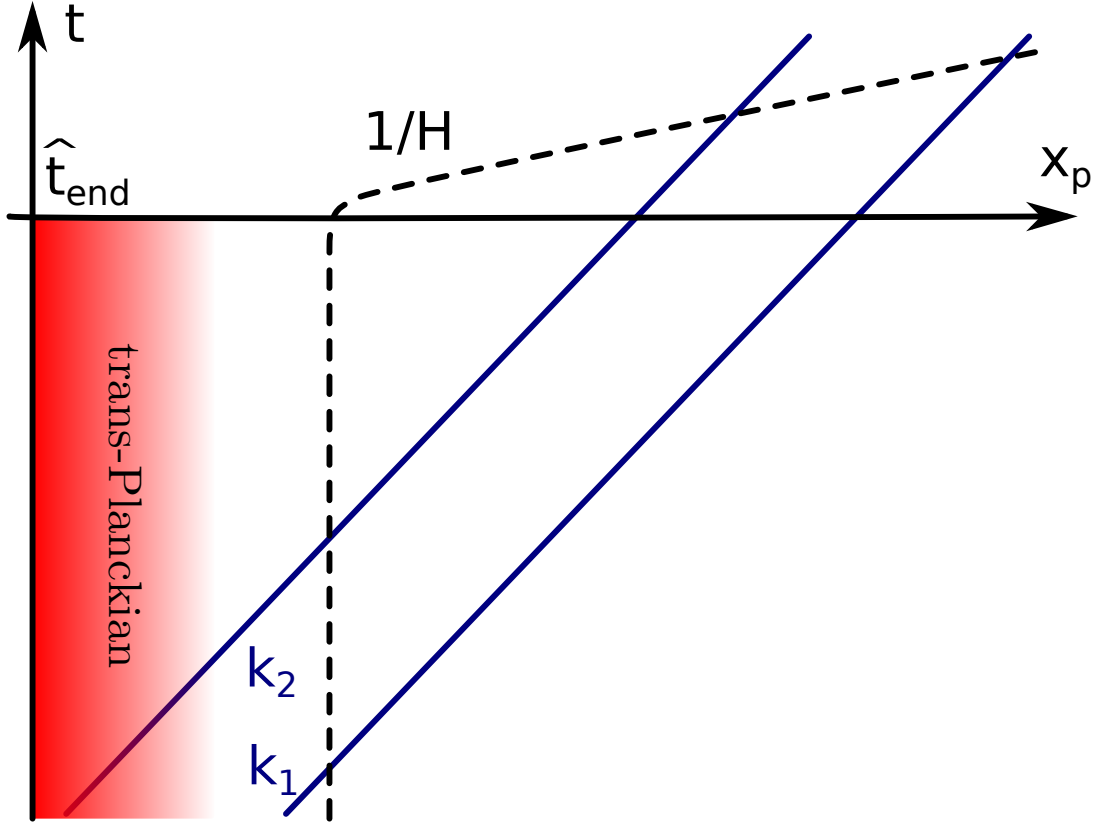


FIG. 6. The transPlanckian problem. The inflationary perturbations are initialized on sub-Planckian scales (or above other UV completion scales).

where

$$v_k = C_+(k) \frac{H}{\sqrt{2k^3}} (1 + ik\tau) e^{-ik\tau} + C_-(k) \frac{H}{\sqrt{2k^3}} (1 - ik\tau) e^{ik\tau} . \quad (34)$$

The quantization condition requires

$$|C_+(k)|^2 - |C_-(k)|^2 = \frac{H^2}{2k^3} . \quad (35)$$

The lowest energy state has $C_- = 0$, which corresponds to the BD state.

The tensor power spectrum for each polarization mode is

$$P_{\gamma}^{+,\times} = |C_+^{+,\times}(k) + C_-^{+,\times}(k)|^2 \left(\frac{H}{2\pi} \right)^2 . \quad (36)$$

$$P_{\gamma} = \frac{2H^2}{\pi^2 M_p^2} [|C_+^+(k) + C_-^+(k)|^2 + |C_+^{\times}(k) + C_-^{\times}(k)|^2] . \quad (37)$$

Note that the k -dependence of $C_{+,-}^{+,\times}(k)$ is determined by high energy physics and is unknown at low energy scales. Thus this k -dependence could in principle tilt the tensor spectrum to the blue end and overwrite the red tilt from decreasing H . This requires

$$\frac{\partial}{\partial \ln k} \{ H^2 [|C_+^+(k) + C_-^+(k)|^2 + |C_+^{\times}(k) + C_-^{\times}(k)|^2] \} > 0 . \quad (38)$$

and the tensor tilt is

$$n_T = -2\epsilon + \frac{d \ln [|C_+^+(k) + C_-^+(k)|^2 + |C_+^{\times}(k) + C_-^{\times}(k)|^2]}{d \ln k} . \quad (39)$$

We would like to make a few comments before closing up this section:

- The non-BD state of tensor modes also opens up possibilities for small field inflation to be consistent with the observed large tensor to scalar ratio $r = 0.2$. However, note that a reason is needed for the scalar sector and the tensor sector to achieve different non-BD coefficients.
- Non-BD initial condition generically introduce non-Gaussianities of the folded shape [37, 38]. The size of the non-Gaussianity is proportional to $\text{Re}[C^-(k)]$ and peaked at the folded limit $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3$. In the squeezed limit $k_3 \rightarrow 0$, the folded non-Gaussianity also blows up more quickly than local shape (however, in the regime under theoretical control, the squeezed limit remains small [39, 40]). Thus the tensor modes generated by non-BD coefficients are non-Gaussian. Despite of the non-Gaussianity in the tensor sector, it is also reasonable to expect a folded non-Gaussianity for the scalar sector. Because if the non-BD issue exists, it is likely not tensor-only unless a reason is provided.

VI. OTHER POSSIBILITIES

There is a huge landscape of inflation models, and also quite a few alternatives to inflation. Some of them may also be able to produce blue n_T . We list a few possibilities here.

A. Inflation: External sources for tensor modes

Besides gravitational waves from quantum fluctuations during inflation, there is a possibility that the dominant gravitational waves are generated by particles or strings [41], or particle states [42, 43] produced in the period inflation. The kinetic energy of the inflaton $\dot{\phi}^2 \sim 2\epsilon H^2 M_p^2$ dumps to the particles, and it is large enough to produce visible tensor spectrum. The emission rate is related to the square of the coupling constant, i.e, for the particles with energy E , rate $\propto E^2/M_p^2$.

One can derive a bound on ϵ from those class of particle production mechanism of sourcing tensor mode. One can estimate the energy density of the classically sourced gravitational waves energy density ρ_{GW} near horizon crossing:

$$\rho_{\text{GW}} < \epsilon M_p^2 H^2, \quad (40)$$

where $\epsilon M_p^2 H^2$ is the kinetic energy density of the inflaton. On the other hand, near horizon crossing, the physical wave length of the tensor modes is of order Hubble scale. Thus

$$\rho_{\text{GW}} \sim M_p^2 (\partial h)^2 \sim M_p^2 H^2 h^2. \quad (41)$$

Thus the tensor power spectrum

$$P_T \sim \langle h^2 \rangle < \epsilon. \quad (42)$$

From the observed tensor to scalar ratio

$$0.2 = r < \frac{\epsilon}{P_\zeta} = \frac{\epsilon}{2 \times 10^{-9}}, \quad (43)$$

We get

$$\epsilon > 4 \times 10^{-10}. \quad (44)$$

From COBE normalization, we get

$$H > 9 \times 10^{-9} M_p \sim 2 \times 10^{10} \text{GeV}, \quad \rho = 3M_p^2 H^2 > 2 \times 10^{-16} M_p^4 \sim (3 \times 10^{14} \text{GeV})^4 \quad (45)$$

Below those scales, no classical source could generate primordial tensor modes.

In some simple models proposed in [41], the produced particles have a time-dependent mass. $M^2 \sim \dot{\phi}^2 t^2$ due to its coupling to the rolling of inflaton. Compared with the fact that inflation has the decreasing Hubble constant, which predict a red tensor spectrum, the increasing mass will afford an explanation of the blue tensor spectrum.

It is also important to note that non-Gaussianities are also produced from the particle production process [44]. The constraints from non-Gaussianity of particle production remains to be tested.

B. Inflation: Beyond slow roll

Based on the slow roll expansion [45], it is recently pointed out that violation of slow roll can generate blue tensor spectrum [46]. In [45], the tensor spectral index is calculated up to second order in slow roll as

$$n_T \simeq -2\epsilon - 2\epsilon^2 - 0.54\epsilon\eta, \quad (46)$$

Thus if the slow roll expansion is extrapolated to $\eta \lesssim -3.7$, blue tensor spectrum is obtained. For example, the possible blue tensor spectrum in the ultra-slow-roll inflation [47, 48] is discussed in [46]. It would be interesting to further examine this possibility without slow roll restrictions.

C. Inflation: Modified gravity

The tensor modes come from the gravity sector. Thus there is no surprise that if gravity is modified, the tensor spectrum could change.

Among other possibilities, massive gravity is one example to achieve this goal. Massive gravity on a time dependent background is a challenge [49]. Nevertheless, there exist viable models [50–52] and application to inflation [53, 54]. We may expect tensor modes with blue spectra in massive gravity: If $m^2 > 0$ for the graviton during inflation, the tensor modes which exit the horizon earlier have more time to roll back to the origin of their mass potential, and thus are suppressed more. As a result, a blue tensor tilt can be generated.

However, the massive gravity in the late universe does not help for the tensor modes. This is because the mass of the graviton is too small. Also, the return-to-horizon effect actually generates a redder spectrum for the gravitational waves [55].

D. Inflation: With space-like condensates

As another interesting class of possibilities, some inflation models come with space-like condensates of fields. For example, space-like gradient of scalars in solid inflation [56] and SO(3) massive gravity [54], or space-like vector fields in Chromo-Natural inflation [57]. Typically, SO(3) spatial rotational symmetry is imposed to keep isotropy. In this class of models, there can be additional tensor components from decomposition of the SO(3) group, other than the graviton. Those components can also contribute to the B-mode power spectrum. For the additional components, there is no NEC type restriction against blue spectra. For example, solid inflation predicts a slightly blue tensor spectrum [56].

E. Matter bounce

As another alternative to inflation scenario, in matter bounce, the universe was in a contracting phase before bouncing back and heating up (see [58] for a review). In matter bounce, the tensor modes are nearly scale invariant but the amplitudes have been too high. Recently, a two-field bounce model is introduced and the tensor modes can be tuned as a parameter [59]. It is thus interesting to see if this model (among other possibilities) fit the current observations and has a potential for blue tensor spectrum.

VII. CONCLUSION AND DISCUSSION

To conclude, we fit the tensor-to-scalar ratio and the tensor spectral tilt with data. The current data is not good enough to test the inflationary consistency relation but nevertheless blue tensor spectra are favored when all 9 bins of BICEP2 data, in combination with the POLARBEAR data, are used.

From theoretical aspects, string gas cosmology predicts blue tensor spectra. However, the tilt is small, at the same order-of-magnitude of scalar tilt. Thus the future experiments targeting to test the inflationary consistency relation can also test this prediction of string gas cosmology. On the other hand, string gas cosmology predicts highly Gaussian density and tensor perturbations. This is unlike the other inflationary mechanisms, where a blue tilt also implies non-Gaussianities.

The simplest model of G-inflation, on the other hand, tilts scalar and tensor power spectra in the same way. Thus with a red scalar spectral tilt, the tensor spectra are also red. The equilateral non-Gaussianity of G-inflation, at $r = 0.2$, is about 30 and close to the current observational bound. The non-Gaussianities should be a model independent

feature for super-inflation type models which generate blue tensor spectrum, because the perturbations have to behave differently from the background to avoid ghosts.

Generalized initial conditions of inflation is left largely unconstrained. However, the generalized initial conditions are also sources of non-Gaussianities. Thus non-Gaussianities in the tensor sector would be a test of those class of models.

Inflationary particle production is another possible source of tensor modes. We derived a bound of inflationary Hubble scale and energy density for this mechanism to work. Non-Gaussianities are also present in the case of particle productions. We hope to investigate the particle production mechanism and its relation between the blue tensor spectra in a future work.

Note added:

Two related works [14, 60] appeared on arXiv on the same day as ours. [60] (see also [61]) overlaps with our Section II, and [14] overlaps with Section V.

The data analysis of tensor tilt is also investigated by [62], [63] and [64], within a few days before/after our paper. Among those papers, [62] and [64] reports a nearly zero central value, with small errorbars $\Delta n_T \sim 0.48$ (BICEP2 only) and $\Delta n_T \sim 0.24$ (BICEP2 + Planck + WP, with running of scalar spectral index). While the following works prefer blue tilt with considerably larger errorbars: [60] (BICEP2 only), our result (BICEP2 + POLARBEAR), [61] (BICEP2 + Planck + WP and BICEP2 x Keck + Planck + WP) and [63] (BICEP2 + Planck + WMAP + BAO).

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